

Quantum Cooperation of Insects

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We investigate the cooperation of two insects who share a large number of maximally entangled EPR-pairs to help them decide whether to execute certain actions. In the first example, two ants must push a pebble, which may be too heavy for one ant. In the second example, two distant butterflies must find each other. In both examples the individuals make classical random choices of possible directions, followed by a quantum decision whether to move or to wait. This combination reflects scarce environmental information and the small brain's limited capacity for complex analysis. With quantum mechanical entanglement the two ants can push the pebble up to twice as far as uncorrelated ants, and the two butterflies need only between 48% and 83% of the classical flight path to find each other.

I) Introduction

Quantum mechanical entanglement is an omnipresent feature in nature, because it is a by-product of interactions. However, with increasing number of degrees of freedom of the entangled systems, the possible modes of coupling to the environment also increase, so that the entanglement occurring between macroscopic systems can easily be swapped into the environment. This is partly countered by the fact that, with increasing complexity of the entanglement, the deviation from classical physics as witnessed by violations of Bell inequalities becomes stronger [1], [2]. Therefore, traces of quantum mechanical correlations might be noticeable even after appreciable contact with the environment. In certain solids this has already been detected, because macroscopic properties like the behavior of the magnetic susceptibility are a proof of entanglement even at finite temperature [3,4].

This opens room to consider the possible effects of entanglement in the biological realm. First suggestions for a role of entanglement between animals have already been made in [5]. There have also been hypotheses of quantum computations taking place in the brain [6]. Indeed, the surprisingly efficient interplay of animals despite little or no communication is reminiscent of the coordinating power of entanglement. One can easily think of a wide range of biological processes, where entanglement would lead to a Darwinian advantage: Entanglement could coordinate biochemical reactions in different parts of a cell, or in different parts of an organ. It could allow correlated firings of distant neurons. And — as shall be the topic here — it could coordinate the behavior of members of a species, because it is independent of distance and requires no physical link. It is also conceivable that entanglement correlates processes between members of different species, and even between living systems and the inanimate world.

Specifically, the evolutionary advantage of entanglement could consist in the following: Since entanglement involves no transfer of information between the entangled parts, but produces effects over arbitrary distances as if information had been transferred [7], it can *eliminate the need* to develop further means of communication.

Nevertheless, entanglement is a fragile relation easily broken in the thermodynamic environment of living systems. It is therefore reasonable to expect that entanglement could have evolved predominantly with such tasks and processes, which can also be accomplished *without* entanglement, but will consume less resources like time or energy, if *supported* by entanglement. In this paper I want to present two such examples. They are based on the entanglement of pairs of spin- $\frac{1}{2}$ particles and deal with the cooperation of two ants and of two butterflies, respectively. I chose such small creatures, because their joint achievements can be so impressive, that it is hard to believe that the limited computational capacity of their nervous systems alone is responsible for them. I will, however, not specify a physical or biological mechanism which carries and protects the systems within an individual which are entangled with similar systems in the other individual, although some remarks will be made in the discussion. We are here mainly interested in how entanglement could be embedded in the stream of sensing, decision making and acting of the individuals. We shall see that one way how entanglement could work is as an "inner voice", which suggests a specific action to the individual, despite a lack of sufficient information for a rational decision. In this sense it could operate like some quantum games, which help the players win even if there is no communication between them [8], giving some of these games properties of "pseudo telepathy" [9].

II) Two Ants Pushing a Pebble

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We look at two ants who must push a pebble towards a certain goal. Each ant j , ($j = 1, 2$) is able to push with a force \vec{f}_j . In order to move the pebble a minimum force f_{min} must be applied. Clearly, if the pebble is too heavy to be moved by any of the ants alone, i.e. $f_{min} > |\vec{f}_1|$ and $f_{min} > |\vec{f}_2|$, the two ants must push simultaneously, and they must push in similar directions.

The two ants go about their task by making a series of *simultaneous* push attempts. However, at each push attempt each ant decides with a probability of $\frac{1}{2}$ whether to push or to rest. This is where we will permit quantum correlations to come in. A push attempt is successful if the force applied to the pebble is larger than the required minimum. Then the pebble will move a little path length proportional to the force and in the direction of the force. The two ants share a large number of pairs of maximally entangled spin- $\frac{1}{2}$ particles in the triplet state:

$$|++\rangle + |--\rangle \quad (1)$$

Each pair is reserved for a specific push attempt. Before a push attempt, an ant must make two decisions:

- Choose the direction β_j of the push. This is a classical decision derived from the sensory input, e.g., the odor emanating from the goal. The chosen direction shall be subject to a probability distribution $w(\beta_j)$. Thereby we simulate that the ants are not clever enough to keep concentrating on the task, or that they may be distracted by other sensory input, or that gusts of wind may temporarily obscure the olfactory information, etc.
- Decide whether to really push at this attempt, or whether to have a little rest. This decision will be made by a projective measurement of the spin of the particle reserved for this attempt along the push direction just chosen. The ant will only push, if the result is "++". This option simulates an ant's need to come to a decision about an action despite a general lack of sufficient classical information, or a general inability to come to an informed decision because of the limited capacity of its brain.

These decisions are made independently by each ant and they are not communicated to the other ant. Of course, *after* the action, each ant could in principle obtain some information about the decisions of the other ant from the effect on the position of the pebble. We assume that this shall not influence an ant's further strategy, because we wish to see the pure effect of the quantum correlations without classical communication between the partners. (Classical communication *after* the fact will be permitted in our next example with the butterflies.)

In case ant j does decide to push, the force it applies to the pebble is

$$\vec{f}_j = s_j \begin{pmatrix} \sin \beta_j \\ \cos \beta_j \end{pmatrix}, \quad (2)$$

where s_j represents the strength of the ant. The direction β_j can range from $-\pi$ to $+\pi$ where $\beta_j = 0$ is the direction straight to the goal.

In a single push attempt there are three possibilities how force can be applied to the pebble. Either by one ant alone, which happens with probabilities $p_{+-} = p_{-+} = \frac{1}{2} \sin^2 \left(\frac{\beta_1 - \beta_2}{2} \right)$. Or by both ants together, which happens with probability $p_{++} = \frac{1}{2} \cos^2 \left(\frac{\beta_1 - \beta_2}{2} \right)$. Knowing that in all three cases the pebble will only move if the force is larger than f_{min} , the expected displacement of the pebble after *one* push attempt is

$$\begin{aligned} \vec{R} = & g \sum_{j=1,2} \int_{-\pi}^{\pi} d\beta_j w(\beta_j) \frac{1}{2} \sin^2 \left(\frac{\beta_j - \beta_{\neq j}}{2} \right) \vec{f}_j(\beta_j) \Theta \left(\left| \vec{f}_j \right| - f_{min} \right) \\ & + g \int_{-\pi}^{\pi} d\beta_1 \int_{-\pi}^{\pi} d\beta_2 w(\beta_1) w(\beta_2) \frac{1}{2} \cos^2 \left(\frac{\beta_1 - \beta_2}{2} \right) \left(\vec{f}_1(\beta_1) + \vec{f}_2(\beta_2) \right) \Theta \left(\left| \vec{f}_1 + \vec{f}_2 \right| - f_{min} \right) \end{aligned} \quad (3)$$

Here, g is the proportionality constant translating applied force to pushed distance in a single push attempt, $\beta_{\neq j}$ refers to the angle not integrated over, and $\Theta(x)$ is the step function whose value is 1 if $x \geq 0$ and 0 otherwise. Since successive push attempts are independent of each other, the expected endpoint after N push attempts is simply $N\vec{R}$.

This quantum mechanical model can be contrasted with a classical one, which is identical, except that the two ants make completely independent decisions whether to push or to rest. We can still imagine these decisions to be based on spin measurements along the chosen directions. But the spins shared by the two ants will be in a fully mixed product state. Then the above quantum mechanical probabilities p_{+-} , p_{-+} , and p_{++} will be independent of the chosen angles, and will have a constant value of $\frac{1}{4}$. Note that, in this classical case, the ants will also be able to push the pebble towards the goal. And we can expect that, as long as the pebble is light enough to be pushed by a single ant, there should be little difference in how far the pebble gets pushed with a given number of attempts. However, as soon as

the effort of both ants is needed to move the pebble, the quantum mechanically entangled ants will become superior. The reason is that especially for small differences between β_1 and β_2 – which are the choices of angles needed to move a pebble too heavy for a single ant – the ants will make the *same* decision of whether to push or to rest up to *twice as often* as the classical ants. Hence, the behavior of the entangled ants will be more coordinated, resulting in fewer futile push attempts. Nevertheless, the behavior of a *single* ant which is quantum mechanically entangled with another one, will be indistinguishable from a classically independent ant.

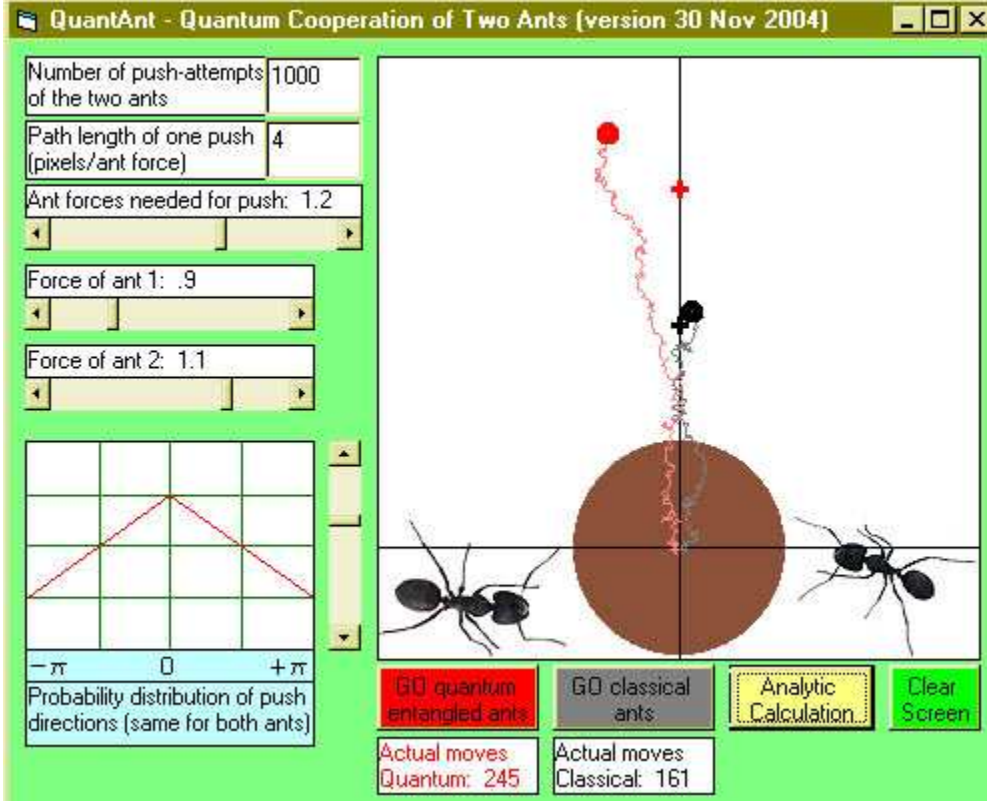


FIG. 1. Typical stochastic paths of the pebble as pushed by quantum entangled as well as by classically independent ants. The diagram on the left shows the probability distribution of push directions given by eq.(4). It can be adjusted from strongly forward peaked ($z = 0$) to completely isotropic ($z = 1$).

Figure 1 shows the result of 1000 attempts of the two ants, for the classical (black) as well as for the quantum case (red). It is a screen shot of a program written to simulate the actual random decisions [10]. The ants must push the pebble to the upper line of the picture. The erratic lines with the fat endpoints show typical paths of the pebble. The crosses indicate the theoretically expected endpoints of the pebble, as given by eq.(3). For the sake of simplicity the probability distribution for the choice of push direction of an ant has been assumed to be the same for both ants and was taken as piecewise linear for positive and negative angles, respectively,

$$w(\beta) = [|\beta(1 - z)| + \pi] [(1 + z)\pi^2]^{-1}, \quad (4)$$

where z is a positive constant between 0 and 1. For $z = 0$ the distribution is triangular with the peak at $\beta = 0$. For $z = 1$ there is no preferred direction.

Figure 2 shows the ratio of the distance the pebble can be pushed by the entangled ants over the distance achievable by the classical ants with the same number of push attempts. Note that these results turn out to be independent of the particular value of z in the probability distribution of directions in eq.(4). The solid line shows the case of two equally strong ants (both can exert a pushing force of 1). Not surprisingly, the superiority of the entangled ants really sets in as soon as the pebble becomes too heavy to be moved by one ant alone. If the pebble requires the maximum force of the two ants, the entangled ants can push the pebble twice as far as the classical ones. The reason is that both ants must push in the same direction. Then the entangled ants will either both push or both rest, while the classical ants will do so only half as often. The more general case of two ants of different strength is shown by the

dashed line. A certain advantage of the entangled ants sets in as soon as the force needed to move the pebble is larger than can be exerted by the weak ant (which here is 0.9), because all efforts by the weak ant alone become futile. If the pebble becomes so heavy that even the strong ant cannot move it alone, the full advantage of the entangled ants sets in.

It is interesting to note that in both scenarios a very small advantage for the entangled ants exists also when the pebble can be moved by any of the ants alone. This is due to the fact that, even for light pebbles, there is a difference in how the pebble gets moved. E.g., for large angles between the chosen directions the forces on the pebble may almost cancel each other and the pebble will not move. Such futile attempts occur with some frequency for the classical ants. But they are rare for the entangled ants, because the $\cos^2(\cdot)$ -term in eq.(3) tends to vanish for such angles. Thus entangled ants profit from making fewer "wrong decisions".

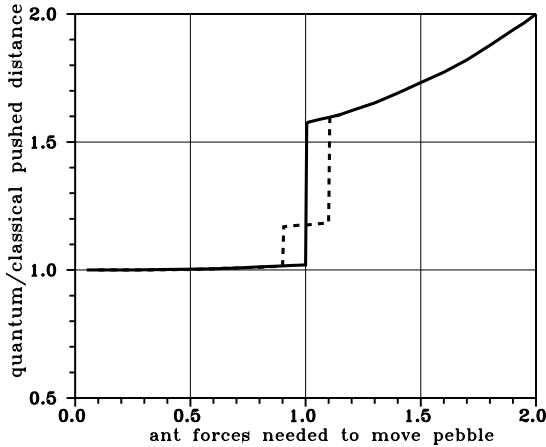


FIG. 2. Ratio of distances the pebble was pushed, of quantum entangled ants over independent classical ants, as a function of the force needed to move the pebble. Solid line: Both ants have force 1. Dashed line: One ant has force 0.9, the other has force 1.1.

III) Two Butterflies Finding Each Other

With certain kinds of butterflies it is known that a male and a female can find each other even when they are initially many kilometers apart. The classical explanation is that each butterfly emanates scent molecules to guide the other one. The huge antennas of a butterfly capture the molecules, permitting it to determine the gradient of the distribution and hence the direction of the origin of the scent. Nevertheless, one may wonder, whether for large distances the classical information contained in the few scent molecules is sufficient to give a butterfly a clear direction where to fly.

In the present scenario we shall assume that this classical information will allow a butterfly only to come up with a probability distribution for the directions in which it should fly. And as in the case of the ants, we assume that the two butterflies share a large number of maximally entangled pairs of spin- $\frac{1}{2}$ particles, of which each butterfly holds one particle. However, here the entangled state will be of the form

$$|+-\rangle - |-+\rangle \tag{5}$$

because we want the butterflies to get the same measurement result, if they project the spins along opposite directions. After all, they are supposed to fly towards each other.

The details of the scenario are as follows:

- (i) The intensity of the scent emanated by each butterfly drops off as $1/r^2$ where r is the distance from the butterfly.
- (ii) The propagation of the scent is very much faster than the speed with which the butterflies fly, so that each butterfly can notice a change of the distance of the other one more or less immediately as a change of intensity of the scent.
- (iii) Each butterfly moves in a sequence of short straight flights of constant length. Before such a short flight the butterfly has to make two decisions in exactly this order:

- Choose a direction for the short flight.
- Decide whether to really do the short flight, or whether to have a little rest.

The first decision is resolved classically. The butterfly chooses the direction for the short flight randomly, but weighted with the probability distribution of directions which it considers appropriate in view of its experience of change of intensity of the scent in the previous short flights. In the model calculations, each butterfly can choose among 16 directions evenly spaced over 2π . In the beginning, this probability distribution is isotropic. After each short flight, the distribution is updated according to a rule explained below.

The second decision comes from a quantum mechanical measurement. The butterfly takes the spin- $\frac{1}{2}$ particle designated for this short flight, and projects it along the chosen direction. If the result is "+", it does the short flight, otherwise it rests until the next short flight is due.

The rule for updating the probability distribution of flight directions now looks as follows. (It is only applied, if the short flight has actually taken place. If, instead, the butterfly has taken a rest, it will retain the probability distribution from before the rest.) The butterfly measures the intensity of the scent of the other one:

- If the increase of the intensity, i.e. the average gradient of the scent along the short flight path, is above a certain threshold, the butterfly judges this to have been a good direction and enhances the corresponding probability weight by the factor $(1+l)$. This direction is then more likely to be chosen again in one of the next short flights. The parameter l can be set between 0 and 1. When it is 0, no learning from experience occurs.
- If the increase of the intensity is below the threshold, the butterfly flies back, because it judges this to have been a bad direction. In addition, it reduces the probability weight of this direction by the factor $(1+l)^{-1}$. This direction is then less likely to be chosen again in one of the next short flights.

The threshold is taken as a certain fraction of the strongest increase of the intensity of the scent encountered in the short flights until then. Therefore, as the butterflies get closer to each other, the threshold will get higher and they will become more discriminating in judging a short flight as having been good or bad.

This quantum scenario can again be compared to a classical one. It is identical to the quantum scenario, except that the decision before each short flight, whether to fly or to rest, is made completely independently by the two butterflies. Each will decide randomly with a constant probability of 0.5 whether to fly or to rest.

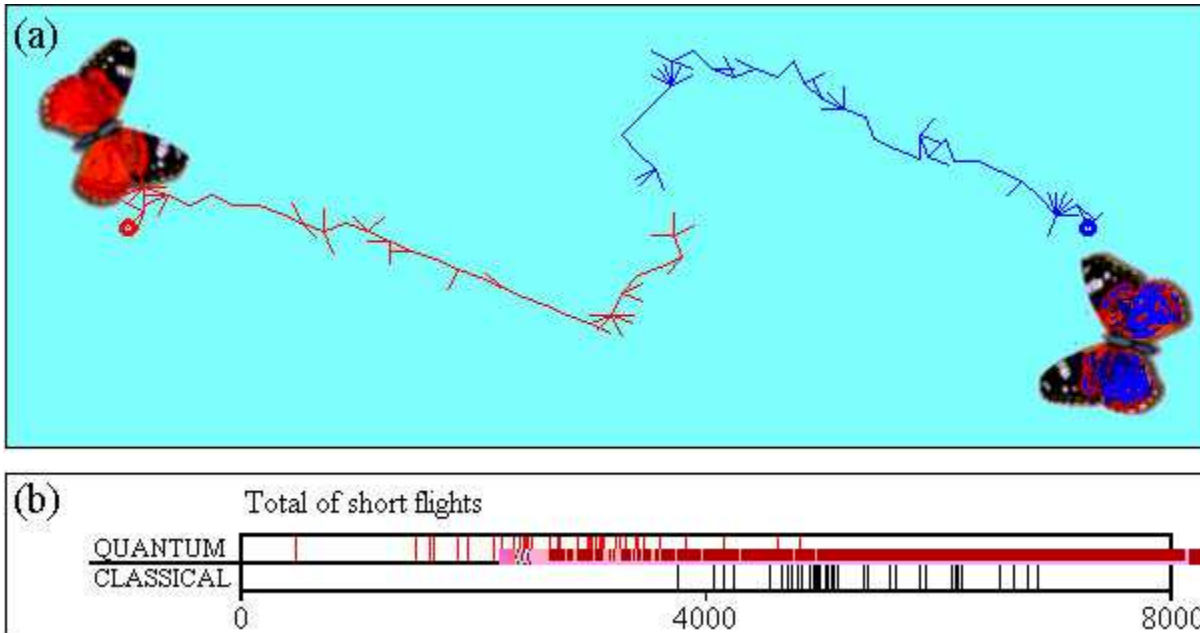


FIG. 3. **(a)**: Typical flight paths of the two butterflies. Initial distance: 1600. Length of one short flight: 40. Learning factor $l = 0.5$. The lines jutting out from the two main lines are short flights after which the respective butterfly had to fly back. Note that sometimes several attempts had to be made to find a good direction. **(b)**: Statistical results of 40 classical and 40 quantum runs, respectively. Initial distance: 1600. Length of one short flight: 5. Learning factor $l = 0.5$. The quantum entangled butterflies needed an average of 2778 short flights to find each other. The classical butterflies needed an average of 5255 short flights.

Figure 3 shows parts of the screenshot of a little program written to analyze the behavior of the two butterflies [10]. The top picture (a) demonstrates actual tracks. The interesting result is the total number of short flights including

the back flights, i.e., the total flight distance the two butterflies had to cover to meet. This is shown in the lower picture (b) which indicates the statistical results of 40 classical and 40 quantum runs, respectively. The red pointers in the upper half indicate the number of short flights of quantum entangled butterflies, the black pointers in the lower half those of classical butterflies. It is noticeable that the quantum butterflies can find each other with much fewer short flights than the classical ones. This is because they decide more often simultaneously to actually do the short flight, if the chosen directions happen to point roughly towards one another. This in turn, gives more often valid short flights, i.e. short flights where the increase of the scent is above the threshold so that neither of them will have to fly back. Also, it leads to a quicker adaptation of the probability distribution of flight directions to favour the current good directions.

The theoretical expectation of the number of short flights needed until the encounter takes place is a laborous exercise and will not be given here. Instead, Figure 4 shows for 40 quantum and 40 classical runs, respectively, the averages and standard deviations of the total flights needed until the butterflies meet, as a function of the learning factor l . The initial distance of the butterflies is 1600. The length of one short flight is 5. The threshold for flying back is that the butterfly does not undo a short flight if the increase of the intensity of the scent of the other butterfly was at least 60% of the strongest increase of the scent found until then. Obviously, the quantum entangled butterflies find each other faster. For a learning factor of $l = 0$ they need only about 83% of the short flights of the classical butterflies, and for $l = 1$ they need only about 48%.

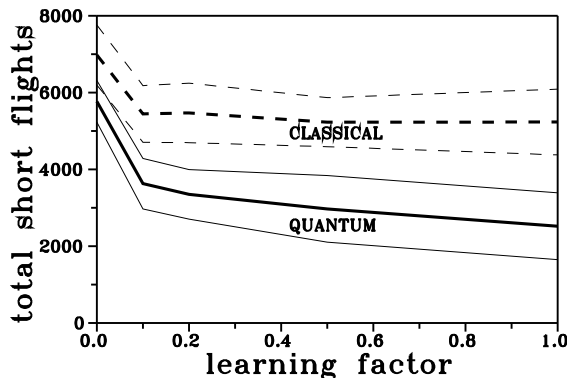


FIG. 4. Total number of short flights needed by the two butterflies to find each other, as a function of learning factor l . Calculated from 40 runs at each l . Solid lines: Average and range of standard deviation for quantum entangled butterflies. Dashed lines: For classical butterflies.

It may be surprising that the quantum entangled butterflies have an advantage over the classical butterflies even for $l = 0$, i.e., when the butterflies don't learn from experience so that for all short flights all directions remain equally likely. This can be understood qualitatively when looking at the probabilities that a butterfly does *not* have to fly back after a short flight. Suppose that for the next short flight the butterflies happen to choose directions pointing exactly at each other. If both happen to really do the short flight, neither will have to fly back, because the increase of the intensity of the mutual scents will be the highest possible. Now, the probability that both will fly is $\frac{1}{2}$ for the quantum case, but only $\frac{1}{4}$ for the classical case. Classically, there is also a chance of $\frac{1}{2}$ that only one butterfly will fly, but in these cases the increase of the intensity of the scent will likely not be high enough and the butterfly will fly back. Altogether, there should thus exist an advantage for the quantum entangled butterflies. And they should have a similar advantage over quite a range of roughly forward pointing directions, the exact angular width of this range depending on the choice of threshold for flying back.

IV) Discussion

Several aspects of the two examples deserve further discussion.

4.1) Communication.

In the example with the ants no communication between the ants is necessary. At each attempt each ant just selects a direction and does or does not push. The momentary position of the pebble, from which some information about the previous decisions of the other ant could be gleaned, has no influence on what choices the ant makes. However, an important communication, which in practice will be olfactory, must permanently happen from the goal to the ants, because the advantage of the entangled ants pushing a heavy pebble results solely from their preference of pushing towards the goal: For similar choices of directions it leads to more frequent occurrences of both ants' *simultaneous* decisions to push, which is the only way to move a heavy pebble.

In the example with the butterflies there is no external goal where both must fly, but each butterfly represents the goal for the other one. Therefore, the classical communication must happen between them, as is effected by each one permanently checking the intensity of the scent of the other one. Without this information the butterflies would have no preference for flying towards each other, because the quantum correlations would also favor directions which lead directly away from each other.

4.2) *Strength of correlations.*

In both examples we have only looked at maximally quantum entangled decisions of the two cooperating insects on the one hand, and compared them to completely independent decisions on the other. Clearly, there is also the possibility of correlations whose strength lies anywhere between, e.g., purely classical correlations which would fulfill Bell's inequality. Such correlations would lead to achievements between those of the two extremes we have investigated, because the strength of the correlations is directly reflected by how far a heavy pebble gets pushed by the two ants, or by how soon the two butterflies find each other, respectively.

4.3) *Critique of the models.*

The assumptions in our simulations are certainly much too simple and much too technical to be found in real insects. It is unlikely that living systems store qubits as spin- $\frac{1}{2}$ particles and that each of these spins is entangled with a similar spin in another living system. It is even more unlikely that in successive decisions an individual resorts to just that spin which is entangled with the spin which the other individual happens to resort to for just the appropriate decision. Such clock-like synchronicity could perhaps be implemented in artificial devices, but would not appear in animals. Also, there is no reason why entanglement should exist between just two, instead of three or more individuals. Nevertheless, our simulations underline that quantum correlations between decisions of cooperating individuals — no matter how these correlations are physically realized — can enhance the cooperative achievements significantly beyond those obtainable with even the fullest exploitation of the available classical information as might be facilitated by neural networks. This is because quantum correlations do contain extra information which is inaccessible to any individual alone but comes to the fore with joint tasks.

4.4) *How is entanglement possible?*

Having made the above criticisms we mention a few results, which might lend support to entanglement within or between living systems, because they address the two important issues of creation and persistence of entanglement. The references are only an entry to the literature.

Let us first look at the creation of entanglement. Entanglement arises automatically from interactions with several possible outcomes [11]. This mode is virtually excluded in our scenarios, because we assumed that the two mutually entangled parts in the insects are protected. The relevant mechanism must therefore work through intermediaries. This has been much studied in the context of quantum computation. One such possibility is entanglement swapping [12]. It requires auxiliary systems which are already entangled, and which then interact with the systems to be entangled. The process can be repeated arbitrarily, but the final entanglement gets more and more diluted. Another possibility is that the systems to be entangled each interact in succession with one external system. This has been done with two atoms passing a cavity one after the other [13], and with two macroscopic Cs-samples, which one after the other were traversed by a beam of light [14]. A related possibility is that the third system interacts independently with the two systems to be entangled, and is then subjected to frequent measurements later on [15]. Here one can also have more intermediate systems. This is a favourable mechanism, because the systems which lie between the two entangled parts in the two insects will be measured permanently by the thermal environment.

Now to the issue of persistence, which means the slow-down of decoherence. That decoherence need not set in immediately has been estimated in a different context for the microtubuli of the brain where quantum states can exist for relatively long periods [6,16]. Recently, it has been found for certain kinds of entangled states that the entanglement of M subsystems, each consisting of many spin- $\frac{1}{2}$ particles, becomes more robust against destructive interactions with the environment, when the number of particles per subsystem increases [17]. The most robust entanglement is obtained when there are only $M=2$ subsystems. (The experiment showing entanglement between two macroscopic samples of Cs-atoms seems to confirm this [14]). For our models this could mean that entanglement between just two living systems is more likely than between three or more systems, and that the entangled systems should have many degrees of freedom, which is certainly the case for bio-molecules. The findings of Zukowski et al [1] and of Mermin [2] point in the same direction. Both works show that the violation of Bell inequalities, and thus the strength of correlations, becomes stronger with higher dimensional systems. A further result, which shows that quantum states need not decohere immediately in a thermal environment, comes from the study of the state of a central spin coupled to a spin bath [18,19]: If there is strong entanglement *within* the spin bath, the initially pure state of the central spin will decohere into a mixed state only slowly.

Even if the questions of creation and protection of entanglement are solved, there is still the problem that in our models sequential and synchronized projections of the entangled spins were needed. For entanglement to work in living systems, one would probably need very massive entanglement, such that it is possible for one animal to blindly probe any tiny fraction of its part of the entangled system and for the other animal to do the same on any other tiny

fraction, and yet a strong correlation of the results should obtain. It is conceivable that, due to global conservation of angular momentum, specific configurations of many spins exhibit such properties. But details remain to be worked out.

V) Conclusion

We have modeled the chain of *sensing, deciding and acting* in the cooperation of two insects of a given species in two different scenarios. We have seen that the quantum mechanical correlation of the statistical decisions of the two individuals is clearly an advantage, because it eliminates the need for more complex classical communication. Nevertheless, it must be realized that actual processes in nature will involve many more parameters than our models. The value of the models is rather instructive because they show that, in moments of hesitation and indecision, an "inner voice" in the form of the result of a quantum measurement can be of great help to an individual in achieving a goal of common interest to the species. Therefore, there may have been evolutionary pressure to rely on quantum entanglement for certain decisions individuals need to make despite insufficient information. It may have been simpler for nature to exploit quantum correlations, which come about quite accidentally all the time, rather than to invent elaborate schemes of improved classical communication.

Acknowledgment

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