Proceedings of the
ICANS-XVI
The 16th Meeting of the
International Collaboration on
Advanced Neutron Sources

ICANS - XVI

May 12 - 15, 2003
ZEUGHAUS (Historic Arsenal)
Convention Center
Düsseldorf-Neuss
GERMANY

Volume II

Edited by G. Mank and H. Conrad
Forschungszentrum Jülich GmbH, Jülich, Germany
Intensive Neutron Beams from a Dense UCN Gas

H. Rauch, E. Jericha

Atominstitut der Oesterreichischen Universitaeten, A-1020 Wien, Austria/EU

Abstract

It is known that high phase space densities of particles can be achieved at low energies only. By a proper phase space transformation to higher energies very high values of beam luminosity may be obtained as is routinely used in all high energy accelerator systems. It is, therefore, obvious that also in the neutron case dense ultra-cold neutrons (UCN) have to be produced to start with. The related slowing down process is not governed by the Liouville theorem. Significant progress has been recently achieved by using solid deuterium as moderator. When such a slowing down mechanism is combined with an intense pulsed neutron source and a proper shutter mechanism, this allows (within a trap) phase space densities related to the peak power of the source, corresponding to UCN-densities up to $10^5$/cm$^3$. This density can be transformed to higher energies by means of fast moving multilayers or single crystals, or by travelling magnetic fields. This method produces highly monochromatic beams reaching intensities beyond $10^9$/s.cm$^3$. Various methods to achieve these goals will be discussed.

1. Introduction

Neutrons with wavelengths above $\lambda > 660$ Å corresponding to velocities below 6 m/s can be trapped in material and magnetic bottles since total reflection occurs at all angles of incidence [1,2]. They are extracted from a thermal or cold moderator of reactor or spallation neutron sources as a small part of the Maxwellian energy distribution. To increase this fraction the gravity effect and rotating turbine blades can be used [3], where in both cases the reduction of energy is accomplished with an increase of the beam divergency since the phase space density is conserved in case of conservative force interaction. The highest density of ultra-cold neutrons has been achieved at the Institute Laue-Langevin with a reported value of about 70 cm$^3$.

New developments in this field are based on special slowing down effects in superfluid helium [4,5,6] and in solid deuterium [7,8,9]. In the case of pulsed reactors or spallation sources the phase space density related to the peak flux can be used which results in significantly increased gain factors [10,8,11]. In distinct cases densities up to $10^5$ cm$^3$ may be anticipated. This could provide a sound basis for new investigations of fundamental properties of the neutron and a starting point for phase space transformation of this density to cold and thermal neutron energies, where beam intensities significantly beyond the existing ones become feasible.
2. Basic relations

At thermal equilibrium with the moderator atoms the neutron density is given as (e.g., [2]):

\[
\rho(v, T)dv = \frac{2\phi_n}{v_T^4} v^2 \exp\left(-\frac{v^2}{v_T^2}\right) dv ,
\]  

(1)

with \( v_T = (2k_B T/m)^{1/2} \). \( \phi_n \) denotes the total neutron flux (\( \phi_n = \int \rho(v, T)dv \)). Figure 1 shows this function for \( T = 293.6 \) K, 50 K and 30 K.

![Figure 1: Neutron densities for three different temperatures.](image)

The related phase space density is given by [12]

\[
n(T) = \frac{d^6N(T)}{dV_p} = \frac{\rho(v, T)}{4\pi} = \frac{\phi_n}{2\pi v_T^4} \exp\left(-\frac{v^2}{v_T^2}\right),
\]  

(2)

where \( N(T) \) represents the neutron number density (\( N(T) = \int \rho(v, T)dv = \phi_n (n^{1/2}/v_T) \)) and \( dV_p = dx \ dy \ dz \ dv_x \ dv_y \ dv_z \) is the phase space element. This function is shown in Fig. 2 together with the corresponding gain factors, which are defined as

\[
G(T_1, T_2) = \frac{n(T_2)}{n(T_1)} = \frac{v_T^4}{v_T^4} \exp\left[ -v^2 \left( \frac{1}{v_T^2} - \frac{1}{v_T^2} \right) \right].
\]  

(3)

The phase space current out of the phase space element in a certain direction \( z \) is denoted as

\[
dJ(T) = \frac{d^6N(T)}{dt} = v_z \frac{d^6N(T)}{dz},
\]  

(4)

and the related luminosity is given as
\[ L(T) = \frac{dI(T)}{dA d\Omega} = \frac{\phi_n}{2\pi} \frac{v_z v^2}{v_T^3} \exp \left( -\frac{v^2}{v_T^2} \right) \frac{dv_z}{v_T}, \]  

(5)

![Figure 2: Phase space densities and gain factors for three temperatures.](image)

where \( dt = dz/v_z, \ dA = dx \ dy, \) and \( d\Omega = dv_x \ dv_y / v^2. \)

A special situation exists for ultra-cold neutrons when we deal with the whole ensemble of neutrons with velocities below \( v_{\text{ucn}}. \) In this case we set

\[ dv_x \ dv_y \ dv_z = v_{\text{ucn}}^3 \]  

(6)

and obtain

\[ L_z = \frac{\phi_n}{2\pi} \frac{v_{\text{ucn}} v_z v^2}{v_T^4}, \]  

(7)

and the related gain factor

\[ G_{\text{ucn}}(T_1 T_2) = \frac{v_T^4}{v_{\text{T}_1}} = \frac{T_1^2}{T_2^2}, \]  

(8)

which gives, for the temperature considered before, \( G_{\text{ucn}}(300,50) = 34.5 \) and \( G_{\text{ucn}}(300,20) = 215. \) It should be mentioned that these gain factors are achieved due to statistical (non conservative) slowing down scattering effects.

3. Phase space transformation of ultra-cold neutrons

It is known that the high phase space density achieved for a certain velocity band can be transformed to other velocity bands by reflection from moving surfaces (turbine blades) or moving multilayers or crystals [3,12,13].

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The velocity in the moving frame is (Fig. 3)

\[ v'_z = v_{z,\text{ucn}} + 2v_c \],

(9)

where \( v_c \) denotes the velocity of the moving part which fulfills in the case of lattice diffraction the Bragg condition for back-reflection \( v_c = \frac{h}{2md} \), where \( d \) denotes the lattice constant. The neutron in the laboratory frame obtains a mean velocity \([11]\) \( v_z = 2v_c \). The width of the reflection curve has to be adapted to the UCN band width \( (2v_{\text{ucn}}) \) by means of a spread of the lattice constant \( \Delta d = v_{\text{ucn}}h/2mv_c^2 \). Under these conditions one gets the phase space density for the up-scattered beam

\[ n_s(T,v_c) = \frac{\Phi_n}{2\pi v_T^4} \exp \left[ -\frac{v_x^2 + v_y^2 + (v_z' - 2v_c)^2}{v_T^2} \right], \]

(10)

and since \( v_c \gg v_{z,\text{ucn}} \sim v_{\text{ucn}} \)

\[ n_s(T,v_c) \approx \frac{\Phi_n}{2\pi v_T^4} \]

(11)

and

\[ L_{sz} = \frac{\Phi_n v_{\text{ucn}} v_z' v^2}{2\pi v_T^4}, \]

(12)
which defines the related gain factors in relation to the density at temperature $T_1$

$$G_s(T_1, T_2) = \frac{n_s(T_2)}{n_s(T_1)} = \frac{v_{t_1}^4}{v_{t_2}^4} \exp \left( \frac{v^2}{v_{t_1}^2} \right).$$  \hspace{1cm} (13)

This function is shown in Fig. 4 and shows remarkably high gain factors especially for higher velocities. The resolution of these beams $\delta v/v = v_{\text{ucn}}/v$ increases also with higher velocities.

![Figure 4: Gain factors for cold and thermal neutron beams when phase space transformed from the UCN regime.](image)

The gain factors also depend on the phase space densities at different moderator temperatures. In Figure 5 we plot the maximal phase space densities provided by moderators at different temperatures and the maximum gain factors achievable for velocity bands $v_{\text{ucn}}/v$ as a function of the mean beam velocity $v$.

![Figure 5: Maximal phase space densities and gain factors for phase space transformed beams with a velocity width $v_{\text{ucn}}/v$.](image)
From these figures we note that remarkable gains are feasible and more efforts should be made to exploit these new possibilities.

4. Experimental realization

Different attempts have been made to verify the forced moderation in solid ortho deuterium and and preliminary results are rather promising [7,8,9]. The pumping option which is feasible at pulsed sources should be tested as well and promises additional gain factors in the order of 50 [11]. Figure 6 shows a proposed set-up combining forced moderation in solid deuterium, using the peak power of the source due to a suitable shutter mechanism, and phase space transformation by means of fast rotating lattices or crystals [15].

![Diagram](image)

Figure 6: Sketch of a facility to produce highly intense cold and thermal neutron beams due to phase space transformation of ultra-cold neutrons.

In order to achieve rather high velocities of the reflecting parts, rotating rather than linearly moving systems have to be used. This introduces an additional constraint and some other shortcomings. In this case the velocity depends on the radius (r) of the rotating wheel which requires a radial dependent lattice constant (see Fig. 3).

\[
v_c = 2\pi r \nu = \frac{h}{md}.
\]

(14)

For a shift of UCN towards a velocity band around 1000 m/s one requires a velocity of the rotating lattice of \(v_c = 500\) m/s. This corresponds to 13 648 rpm for a wheel with a radius of
35 cm. This requests a mean lattice constant of \( d = 3.94 \) Å varying for a 5 cm vertical beam size from \( d_{\text{min}} = 3.72 \) Å to \( d_{\text{max}} = 4.26 \) Å. This gives a velocity spread of about 13%. Additionally, a mosaic spread of about 0.6% would be required to reflect all UCNs. The number of reflecting units depends on the time needed for UCN to diffuse into the reflecting volume. Taking into account the luminosity of UCNs (Eq. (7)), a mean velocity \( \bar{v}_{\text{ucn}} = 3 \) m/s, and a reflecting volume of 5x5x5 cm\(^3\), the filling time is about \( \tau_f = 15 \) ms, which is considerably larger than the time for one rotation of the wheel (\( \tau_r = 4.4 \) ms). This means that even when only one reflecting crystal is mounted the filling ratio is only \( \tau_f / \tau_r < 1 \). Larger wheels would lead to a higher monochromaticity and would allow pulsed monochromatic neutrons but make the mechanical part more difficult.

**Discussion**

These calculations and recent test measurements have shown that UCN densities can be increased by orders of magnitudes due to moderation in solid deuterium and by using a pumping option at pulsed sources to fill a rather large UCN trap. Densities beyond \( 10^4 \) cm\(^3\) are feasible and then phase space transformers can produce monochromatic neutron beams of high intensity. It seems to be worthwhile to exploit this path in more detail because it has the potential to open new horizons for neutron research.

This work has been supported by the Austrian Science Foundation (project F1513) and the AUSTRON spallation source project.

**References**

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