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Physica B 241–243 (1998) 1207–1209

**PHYSICA** B

## A proposal to visualize magnetic domains within bulk materials

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### Abstract

The recently proposed neutron magnetic tomography, an extension of the three-dimensional neutron depolarization technique, is considered. The new method allows visualization of magnetic domains within bulk materials. In concept it is similar to X-ray tomography but the analysis is much more involved because of the tensorial character of neutron spin rotations. An improved iterative numerical procedure is presented which fully accounts for the non-Abelian properties of the spin rotation. The method is applied successfully to the reconstruction of domain structures from simulated data of schematic examples. The experimental feasibility is discussed. © 1998 Elsevier Science B.V. All rights reserved.

**Keywords:** Neutron depolarization; Tomography; Polarization analysis

The magnetic domain structure of ferromagnetic materials is of great interest in the material sciences. There are several well-established techniques for the visualization of the magnetic domain structure at the surface [1,2]. However, there is no standard procedure to reveal single domains and their magnetization within bulk materials. The recently proposed neutron magnetic tomography (NMT) appears to be a suitable tool to achieve this goal [3,4]. The first numerical studies were very promising [3,4].

NMT is based on the three-dimensional neutron depolarization technique [5,6], where one measures the change of the polarization of the neutron beam due to the magnetic interaction experienced by the neutron during its motion through the sample.

Within the semiclassical spin rotation formalism [5,6] the equation of motion of the actual polarization  $\mathbf{P}$  is given by

$$\frac{d}{dz} \mathbf{P}(z) = \mathcal{A} \mathbf{P}(z), \quad (1)$$

where  $z$  is the coordinate along the beam direction. The matrix  $\mathcal{A}$  represents the cross product with the magnetic field  $\mathbf{B} = (B_x, B_y, B_z)$ ,

$$\mathcal{A} = \frac{\gamma}{v} \begin{pmatrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{pmatrix}. \quad (2)$$

Here,  $\gamma$  is the gyromagnetic factor and  $v$  is the neutron velocity. Albeit Eq. (1) is valid only for a homogenous field we apply it to the whole beam line. Thus, we implicitly assume that the effect of the Bloch walls on the beam polarization can be

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neglected. The formal solution of Eq. (1) is of the form

$$\mathbf{P}(z_1) = \mathcal{D}\mathbf{P}(z_0) = \exp\left(\int_{z_0}^{z_1} \mathcal{P} dz' \mathcal{A}(z')\right) \mathbf{P}(z_0), \quad (3)$$

where  $z_0$  and  $z_1$  characterize the entrance and the exit point of the neutron beam respectively. The depolarization matrix  $\mathcal{D}$  contains information about the magnetic structure along the beam line. The path ordering  $\mathcal{P}$  takes into account the nonvanishing commutators of  $\mathcal{A}(z)$  and  $\mathcal{A}(z')$  at  $z \neq z'$ .

NMT is essentially a tomographic extension of this technique, where a specific plane of the considered specimen is scanned by neutron beams of different directions. The method is similar to the X-ray transmission tomography but the tensorial character of  $\mathcal{D}$  makes the analysis much more involved. In the absence of an analytical solution we have developed iterative numerical procedures. Here, we present a refined algorithm which takes into account the path-ordering.

The basic ansatz is the factorization of  $\mathcal{D} = \mathcal{L}\mathcal{K}$ , where  $\mathcal{L}$  contains the pure line integral,

$$\mathcal{L} = \exp\left(\int_{z_0}^{z_1} dz' \mathcal{A}(z')\right), \quad (4)$$

and  $\mathcal{K}$  provides the correction for the path ordering. Standard radon transformation [7] of the tomographic set of  $\mathcal{L}$  matrices yields the distribution of the magnetic field  $\mathbf{B}$  [3,4]. To account exactly for the path ordering we evaluate for a given magnetization distribution for each beam  $\mathcal{D}_{\text{th}}$ ,  $\mathcal{L}_{\text{th}}$  and  $\mathcal{K}_{\text{th}} = (\mathcal{L}^{-1})_{\text{th}} \mathcal{D}_{\text{th}}$ . From the experimental  $\mathcal{D}$ , one can then obtain  $\mathcal{L}_{\text{ex}} = \mathcal{D} \mathcal{K}_{\text{th}}^{-1}$ . Application of the radon transformation to  $\mathcal{L}_{\text{ex}}$  yields an improved magnetization distribution which can again be used to calculate new  $\mathcal{D}_{\text{th}}$  and  $\mathcal{L}_{\text{th}}$  for the next iteration.

We have applied this procedure to several examples with simulated data and found excellent convergence as long as the pixel size does not exceed a certain value. In Fig. 1 the convergence is illustrated at a typical example with a  $20 \times 20$  pixel resolution. The quantity  $\Delta$  is a measure of the quality of reproduction (cf. Refs. [3,4]), where  $10^{-5}$

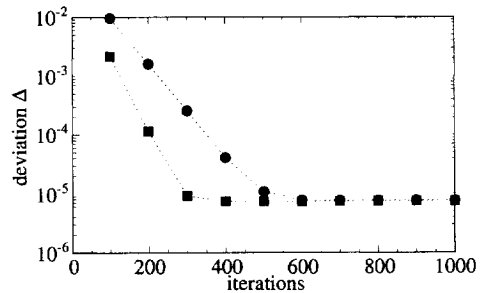


Fig. 1. Convergence of the iteration procedure for different pixel sizes,  $0.8 \mu\text{m}$  (circle) and  $1.2 \mu\text{m}$  (square) for a specific example with a  $20 \times 20$  grid.

represents perfect agreement within the numerical accuracy.

The convergence of an iterative procedure depends strongly on the starting point. In previous work we started from the assumption  $\mathcal{K}^{(0)} = 1$  which is not satisfactory for larger pixel sizes. To improve the starting values, we propose to measure the  $\mathcal{D}$  of each beam line in the forward (F) and backward (B) direction. Thus, one obtains an improved starting value of the  $\mathcal{L}$ -matrix,  $\mathcal{L}^{(0)} = (\mathcal{D}_{\text{F}} + \mathcal{D}_{\text{B}})/2$ , where the lower-order commutator terms are already eliminated. The corresponding  $\Delta$ -values of the starting configuration are one to two orders of magnitude smaller and accelerate the convergence considerably.

A crucial question for the applicability of NMT is the beam time required to perform such measurements with reasonable statistics. First estimates, assuming the parameters of the high flux reactor at ILL and considering beam diameters of about  $10 \mu\text{m}$ , leads to a typical total measurement time (1500 beams, statistics about  $10^3$ ) of about 400 h for a conventional set-up. However, using recently developed focusing neutron optical devices would reduce the required measuring time to about 10 h. Thus, NMT measurements seem to be within experimental reach with the best neutron optical techniques presently available.

The work has been supported by *Fonds zur Förderung der Wissenschaftlichen Forschung, Österreich*, project number P10969-PHY.

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