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Wave–particle duality and quantum erasure in polarized–neutron interferometry

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Abstract

An interference experiment with polarized neutrons is proposed to check the duality relation between the fringe visibility and the which-way information for massive fermions with macroscopic spatial separation of the interfering sub-beams. The connection between polarization-sensitive post-selection of sub-ensembles and the concept of quantum erasure is discussed. This neutron experiment will supplement those recently performed with photons and atoms. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Wave–particle duality (WPD) dates back to the very early days of quantum mechanics, to Einstein's seminal paper on the photoelectric effect [1], and is a striking manifestation of Bohr's principle of complementarity [2,3]. The familiar phrase "each experiment must be described either in terms of particles or in terms of waves" emphasizes the extreme cases and disregards the intermediate situations, in which particle aspects and wave aspects are present simultaneously. Theoretical investigations [4,11], supplemented by a few experimental studies [12–14], have led to a quantitative formulation of WPD, namely the duality relation of Eq. (3) below. Very recently, the validity of the duality relation was tested with a Mach–Zehnder-type *photon interferometer* [15] and an *atom interferometer* [16,17] that mimics some of the essential features of a Young double-slit setup. The experiment we are proposing here will test the duality relation in the realm of *neutron interferometry*.

Since its invention in 1974 [18] perfect crystal neutron interferometry has become a powerful and in many respects unique tool to test and to demonstrate fundamental principles of quantum mechanics with massive particles on a macroscopic space–time scale. This is particularly true if the spin-$\frac{1}{2}$ property of the neutron is explicitly taken into account as it will be the case in our proposal. The list of successfully performed experiments belonging to the spinor character of the neutron

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comprises topics like the first explicit verification of the 4π-periodicity of spinor wave functions [19–21], demonstration of the quantum mechanical principle of linear superposition of states [22,23], macroscopic quantum beating of the neutron wave function [24], multiple photon exchange between the neutron and an applied electromagnetic field [25], neutron interferometric separation of geometric and dynamical phases [26], as well as interferometric [27] and polarimetric [28] verification of the topological character of the scalar Aharonov–Bohm effect. A general state-of-the-art survey about neutron interferometry is found in Refs. [29,30]. Unlike any other matter wave interference technique developed thus far with either electrons, atoms or ions, perfect-crystal neutron interferometry allows a coherent separation of quantum states of massive particles over macroscopic distances of several centimeters followed by subsequent coherent recombination. It is therefore ideally suited to test the validity of the duality relation.

For a quantification of WPD we need quantitative, measurable characteristics for the wave aspects and the particle aspects. The former are naturally quantified by the *visibility* $\mathscr{V}$ of the observed interference fringes. How about the latter? Another familiar phrase, "a wave is in both arms of an interferometer simultaneously, a particle is in one arm or the other", gives a hint. The better we can tell which way has been taken by a particular neutron, the more pronounced are its particle aspects. Their quantification is, accordingly, based on the *likelihood* $\mathscr{L}$ of guessing the way right. Since a random guess gives a likelihood of $\mathscr{L} = \frac{1}{2}$, whereas $\mathscr{L} = 1$ indicates that we know the way with certainty, the actual which-way (WW) *knowledge* $\mathscr{K}$ is given by

$$\mathscr{L} = \tfrac{1}{2} + \tfrac{1}{2}\mathscr{K} \quad \text{with } 0 \le \mathscr{K} \le 1. \tag{1}$$

The value of $\mathscr{K}$ depends on the "betting strategy" we are employing; the optimal strategy maximizes $\mathscr{K}$ and this identifies the *distinguishability* $\mathscr{D}$ of the ways,

$$\mathscr{D} = \max\{\mathscr{K}\}. \tag{2}$$

In an asymmetric interferometer, one way is more likely than the other to begin with, so that we would know something about the way beforehand; WW information of this kind is fittingly called *predictability* (denoted by $\mathscr{P}$), and the inequalities $\mathscr{P} \le \mathscr{K} \le \mathscr{D}$ state an obvious hierarchy because WW detection cannot decrease the betting odds.

The central quantitative statement about WWD is the *duality relation*

$$\mathscr{D}^2 + \mathscr{V}^2 \le 1 \tag{3}$$

that has been found recently [10,11]; there are examples, in which the equality sign holds. The implied statement $\mathscr{P}^2 + \mathscr{V}^2 \le 1$ has been known for some time, implicitly or explicitly, in a variety of physical contexts [4–9,12–14]. All experimental tests refer necessarily to the version

$$\mathscr{K}^2 + \mathscr{V}^2 \le 1 \tag{4}$$

with the knowledge $\mathscr{K}$ optimized under the constraints of the setup, hoping for $\mathscr{K}_{max} \approx \mathscr{D}$. In the experiment we are proposing, we will usually have $\mathscr{P} = 0$ and it should be possible to extract (almost) all the WW information that becomes potentially available, so that the acquired WW knowledge $\mathscr{K}$ is expected to come very close to its theoretical upper bound $\mathscr{D}$.

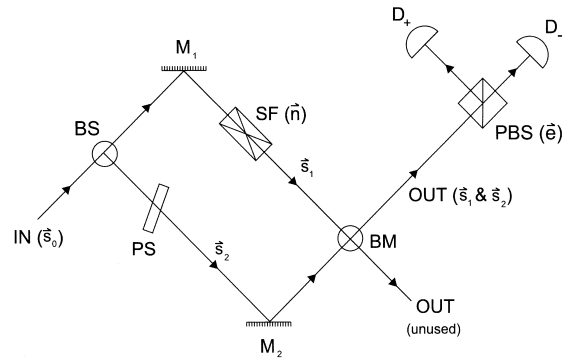

Fig. 1. Schematic sketch of the proposed neutron interferometer with which-way marking (see text).

2. Scheme of the experiment

Fig. 1 shows a schematic sketch of the proposed experiment. It is the neutron analog of the photon experiment of Ref. [15]. A standard Mach–Zehnder-type neutron interferometer is supplemented by devices for (i) encoding WW information in the neutron's polarization and (ii) extracting WW knowledge later on. For the sake of brevity here we are concentrating on the general concepts and not on the actual technical realization.

In the setup of Fig. 1, polarized neutrons enter from the left at the beam splitter BS. After being reflected at one of the mirrors (M_1 or M_2) the amplitudes are recombined at the beam merger BM. The neutrons exiting at the symmetric output port are eventually counted by the detectors D_+ and D_-. Those emerging at the asymmetric output are not used for the measurement itself, only for collecting control data.

2.1. Determining the fringe visibility $\mathscr{V}$

The phase shifter PS in the lower way (BS–M_2–BM) enables us to change the relative phase ϕ of the two partial amplitudes, and so we can scan through the interference pattern. The total count rate of D_+ and D_- will be proportional to $1+\mathscr{V}\cos\phi$, where $\mathscr{V}$ is the fringe visibility. Since we are observing in the symmetric output, the maximal value $\mathscr{V}=1$ is achievable if the partially transparent mirrors that are used for the BS and the BM have identical properties.

2.2. Labeling the neutrons and acquiring which-way knowledge $\mathscr{K}$

In the upper way (BS–M_1–BM) a spin flipper SF can change the initial polarization to another one, so that the neutrons acquire a polarization label that marks their way through the interferometer. This information is read out with the aid of the polarizing beam splitter PBS that sorts the neutrons, so that D_+ counts the neutrons polarized in the $+\boldsymbol{e}$ direction and D_- those polarized in the (orthogonal) $-\boldsymbol{e}$ direction. Here, the unit vector $\boldsymbol{e}$ characterizes the PBS.

Table 1

	D_+	D_-
M_1-way	$1+\boldsymbol{e}\cdot\boldsymbol{s}_1$	$1-\boldsymbol{e}\cdot\boldsymbol{s}_1$
M_2-way	$1+\boldsymbol{e}\cdot\boldsymbol{s}_2$	$1-\boldsymbol{e}\cdot\boldsymbol{s}_2$

The Pauli vector $\boldsymbol{s}_0=\langle\boldsymbol{\sigma}\rangle_0$, which is the expectation value of the spin vector operator $\boldsymbol{\sigma}$ in the initial state, specifies the spin state of the neutrons entering the interferometer at the BS. Neutrons coming along the M_1-way pass through the (unitary) SF which rotates their spin vector around the axis $\boldsymbol{n}$ and so turns $\boldsymbol{s}_0$ into $\boldsymbol{s}_1$; the neutrons of the M_2-way have $\boldsymbol{s}_2=\boldsymbol{s}_0$. If no depolarization occurs, the lengths of $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$ remain the same (identical to the length of $\boldsymbol{s}_0$) and they differ only in their direction.

In general, the count rates of D_+ and D_- will be different for the two ways. Their relative sizes are summarized in Table 1.

The experimental values corresponding to Table 1 entries are obtained by measuring the count rates in each row with the other way blocked.

The betting odds for guessing the way right are most favorable when we always bet on the way that contributes most to the probability of triggering the detector that actually fired. This results in a measured WW knowledge given by [10,11]

$$\mathscr{K}=\frac{|r_{1+}-r_{2+}|+|r_{1-}-r_{2-}|}{r_{1+}+r_{2+}+r_{1-}+r_{2-}}=\frac{1}{2}|\boldsymbol{e}\cdot(\boldsymbol{s}_1-\boldsymbol{s}_2)| \tag{5}$$

where, for example, r_{1+} is the count rate of D_+ for M_1-way neutrons. Clearly, theory predicts that $\boldsymbol{e}$ should be chosen parallel to $\boldsymbol{s}_1-\boldsymbol{s}_2$ for maximal WW knowledge, and that $\mathscr{K}=1$ is only possible if (i) $\boldsymbol{s}_1=-\boldsymbol{s}_2$ and (ii) both are unit vectors. In operational term: The neutrons must be fully polarized in both ways and these polarization states must be orthogonal.

For the equal sign to hold in Eq. (3), condition (ii) suffices. It is then possible to study the trade-off between fringe visibility and WW knowledge under ideal circumstances.

2.3. Post-selection, quantum erasure

The polarization-sensitive detection of the neutrons sorts them into two sub-ensembles. Depending on the choice for the characterizing unit vector $\boldsymbol{e}$, different sortings are possible. For example, the WW sorting just described is given by $\boldsymbol{e}\|(\boldsymbol{s}_1 - \boldsymbol{s}_2)$. Inasmuch as the choice of sorting is undecided until the neutrons reach the PBS in the setup of Fig. 1, that is, until they have left the interferometer proper, the sub-ensembles are *post-selected*.

A sorting of a particularly interesting kind has features in common with *quantum erasure* (QE) [31,32]. It identifies sub-ensembles that exhibit perfectly visible fringes, even when the fringe visibility $\mathscr{V}$ of the unsorted totality of counted neutrons is reduced, possibly to $\mathscr{V} = 0$. This is achieved by setting $\boldsymbol{e}$ equal to the unit vector $\boldsymbol{n}$ that specifies the direction around which the SF rotates the neutron's spin vector.

It is clear that intermediate situations are realized by choosing $\boldsymbol{e}$ different from both the WW choice and the QE choice. This amounts to an experimental realization of *partial QE*.

It is remarkable that the initial polarization $\boldsymbol{s}_0$ is irrelevant for QE. Irrespective of the length and the direction of $\boldsymbol{s}_0$ the sorting characterized by $\boldsymbol{e} = \boldsymbol{n}$ always identifies two sub-ensembles that have fringes with unit visibility. In the extreme case of totally unpolarized neutrons (that is $\boldsymbol{s}_0 = 0$) the SF does not produce a WW label, but will reduce the fringe visibility to zero if the rotation angle is 180°. The fringes are then lost, but no WW information is available [$\mathscr{D} = \mathscr{V} = 0$ in Eq. (3)]. Nevertheless, one can do the QE sorting and recover the fringes. An experimental demonstration of the latter property of QE would confirm that the availability of WW information is not a precondition for successful QE.

3. Experimental essentials

There is no room to present the details of the experimental setup that will be required to realize our proposal. Hence, we completely omit a description of the interferometer itself and focus instead on the methods to prepare the input and to analyze the final neutron state, since they are of particular importance for such kind of experiment.

3.1. Input state preparation

The extremely narrow angular width of perfect-crystal Bragg or Laue reflections enables us to exploit the spin-dependent birefringence of neutrons upon passing through prismatically shaped magnetic fields. The angular separation of the two spin states of an initially unpolarized beam of neutrons with energy E which passes at an asymmetry angle ε (see Fig. 2) through a magnetic field prism of apex angle Φ is given by [33]

$$\delta = \frac{2\mu B}{E} \frac{\sin(\Phi)}{\cos(\Phi) + \cos(2\varepsilon)}. \tag{6}$$

Recently, we could develop for the first time a polarizer of this kind consisting of a permanent magnetic yoke with prismatically shaped air gap of 1 cm height and apex angle $\Phi = 116°$. Without producing any heat a field of about 0.9 T is achieved. Fig. 2 shows the angular splitting measured with a high-resolution double-crystal small-angle camera for symmetric passage through one and two of these field prisms. Owing to the influence of stray fields the experimentally observed

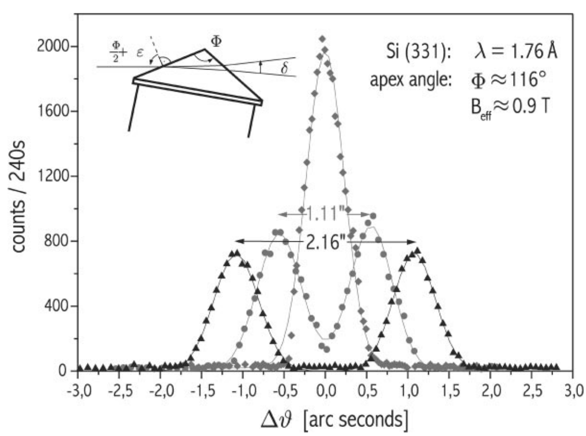

Fig. 2. Angular splitting of the two spin states upon transmission through *one* (●) and *two* (▲) magnetic field prisms. The central curve (◆) displays the resolution curve of the double-crystal small-angle instrument.

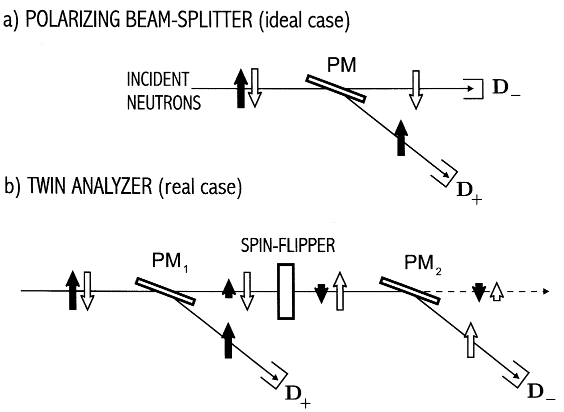

Fig. 3. (a) Ideal polarizing beam splitter (PBS) with 100% reflectivity of the spin-*up* state. (b) Approximate realization of a PBS by placing a high performance spin flipper between two polarizing mirrors with reduced reflectivity

splitting is about 20% smaller than theoretically expected. Nevertheless, it is sufficiently large compared to the reflection width of perfect Si crystals to guarantee fully polarized beams inside the interferometer.

3.2. Spin state analysis

WW detection crucially depends on the quality of the polarization sensitive detection. In the ideal case a polarizing beam splitter sorts each neutron according to its spin state either in detector D_+ or D_- (Fig. 3). Evidently, the reflectivity of such an ideal beam splitter must reach a value very close to 100% in order to avoid a contamination of the transmitted beam with the "wrong" spin state. At least for cold neutrons the performance of supermirror polarizers is quite close to this ideal situation. But even in the case of reduced reflectivity a combination of a pair of such imperfect polarizers should allow for an almost perfect spin-dependent neutron sorting, provided a spin-flip device with an efficiency close to 100% is placed between them, as indicated in Fig. 3. There is no particular difficulty in fabricating neutron spin flippers of the required quality, but it is essential that the two polarizers neither attenuate nor depolarize the transmitted beam. Then a low reflectivity only causes a poor neutron economy, but it has no influence on the significance of the spin state sorting.

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