From Falling Cats to the Geometric Phase in Neutron Interferometry

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Have you ever wondered why a falling cat always lands on her feet? Apparently, the problem of the cat is that due to angular momentum conservation there seems to be no way for the cat to right herself. However, by changing her shape she can affect a rotation as a whole, a geometric effect. In the quantum world, such geometric effects give rise to additional phase factors depend only on the way the system evolves. In particular, we have investigated the geometric phase associated to the neutrons' path in an interferometer.

The question why a falling cat succeeds (almost) always to land on her feet even when starting in an upside-down position has puzzled physicists for a long time. From the basic principles of mechanics angular momentum conservation should hinder the cat to change her orientation while falling. However, she can re-orient herself without any particular problems (Figure 1). Only recently, this has been identified as a geometric effect, where the deformation of the shape of a body has immediate consequences onto its orientation [1]. The cat twists and bends her body in a particular cyclic way in order to obtain a 180-degrees "phase-shift". In other words, it varies her internal degrees of freedom in order to change an external parameter. A detailed examination shows that the non-trivial geometrical structure of the shape space (the set of all possible shape states) is responsible for this effect.

Similarly, in quantum mechanics the system's underlying geometry becomes apparent in an additional phase factor [2]. For example, the neutrons' spin state can be manipulated by external magnetic fields. By varying the magnetic fields the spin state traces out a path on a sphere, its state space, and accumulates a phase factor. If so called *parallel transport conditions* are satisfied the accumulated phase is purely geometric. It is proportional to the area encircled by the evolution path and does neither depend on energy nor on the evolution period like its dynamical counterpart. In the cat-example the parallel transport conditions are naturally given by the angular momentum conservation law.

In our experiment we exploit the fact that the possible paths in a neutron interferometer can be represented on a sphere like spin states. The up (down) spin corresponds to the neutron taking the upper (lower) path at a beamsplitter. Different states can be reached by changing the weighting of the two subbeams or by altering their relative phase. A variation of these parameters leads to a path traced out on the sphere and, therefore, to a geometric phase.

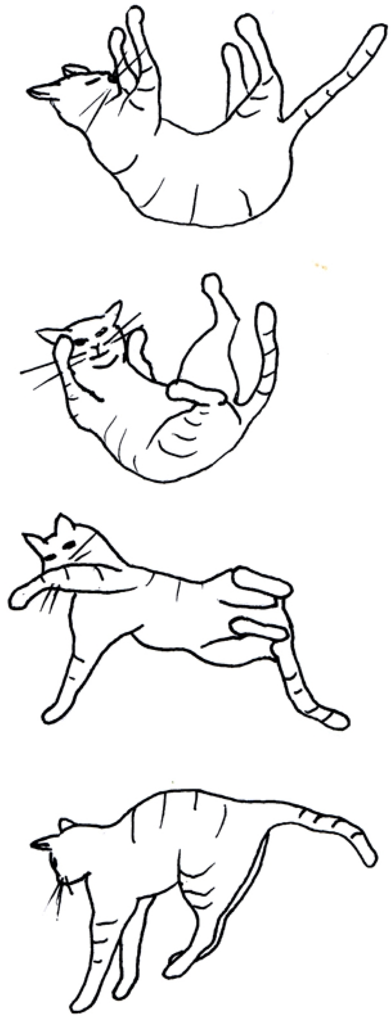

FIG. 1: Falling cat always lands on her feet.

In the double-loop interferometer (Figure 2) the geometric phase generated in the second loop relative to the reference beam $|\psi_r\rangle$ is measured. With a particular choice of the phase shifting slabs and the transmission coefficient of the attenuator the action of the phase shifter on the state in the second loop $|\psi_2\rangle$ does not change its global phase. Only the internal phase relation of the state is altered, i. e. $|\psi_2\rangle \mapsto |\psi_2'\rangle$ with $\arg\langle\psi_2|\psi_2'\rangle = 0$

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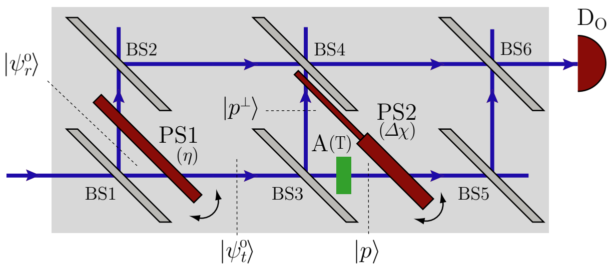

FIG. 2: Experimental setup. The beam reflected at the first beam splitter $|\psi_r^0\rangle$ serves as a phase reference to measure the phase change of the state in the second loop $|\psi_2\rangle \equiv \alpha|p\rangle + \beta|p^\perp\rangle$. By a rotation of PS2 the internal phase relation of $|\psi_2\rangle$ changes, $|\psi_2\rangle \mapsto |\psi_2'\rangle$, but due to the different thicknesses of PS2 the phase difference between two neighbouring states always vanishes, $\arg\langle\psi_2|\psi_2'\rangle = 0$.

and we say that the state is parallel transported. However, a change in the phase $\Phi = \arg\langle\psi_r|\psi_2\rangle$ relative to the reference beam has been observed in an neutron interferomter experiment at the S18 beamline [3]. What's more, this phase can be deduced directly from the surface enclosed by the path on the sphere in Figure 3. In Figure 4 this theoretical line is shown together with the measured values, the consistency demonstrates that the measured phase is geometric and proportional to the solid angle as seen from the center.

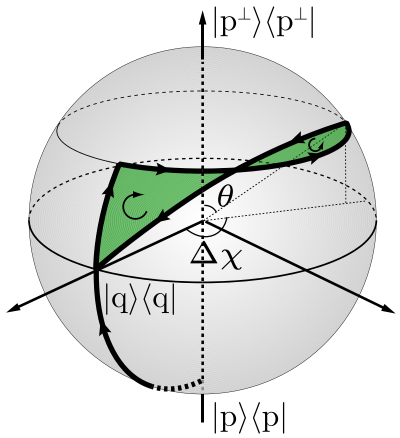

FIG. 3: Each state in the second loop can be depicted as a point on a sphere, where $|p\rangle$ and $|p^\perp\rangle$ are the basis states for finding the neutron either in the upper or the lower path, respectively.

Let us finally return to the falling cat: At a first sight a net rotation is not allowed simply by angular momentum conservation. Similarly, by changing only the internal phase relation in the second loop and not the phase relative to the reference beam we do not expect to observe a net phase change. However, due to the underlying geometry in these problems we observe an additional geometric phase and the cat safely lands on her feet.

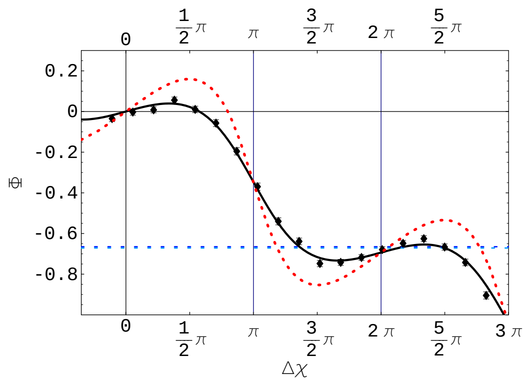

FIG. 4: Measured geometric phase. For the fitted curve (solid line) systematic deviations stemming from the finite coherence of the beams are taken into account [3]. The dotted line indicates the theoretical prediction.

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